HDY-003-016303 Seat No. ________

M. Sc. (Sem. III) (CBCS) Examination

November / December – 2017

Mathematics : 3003

(Number Theory 1)

(Old Course)

Faculty Code : 003

Subject Code : 016303

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are 5 questions in this paper.

(2) All questions are compulsory.

(3) Each question carries 14 marks.

1 Fill in the blanks : (Each question carries **2** marks)

(i) If p is a prime number and n is a positive integer then the number of positive integers relatively prime to p^n is ....

(ii) If p and q are distinct primes then $p^m q^n$ has .... positive divisors. ($m, n \in \mathbb{N}$)

(iii) If p is a prime of the form 4k+3 then $x^2+1 \equiv 0 \pmod p$ has .... solutions.

(iv) If p is a prime number and n a positive integer then the number of positive divisors of p^n = ...

(v) If n = 200 × 202 then $\varnothing(n)$ = ...

(vi) If p is a prime number and p does not divide a then $a^{p-1} \equiv 1 \pmod p$. This theorem is called .... theorem.

(vii) If m divides ab and m and a are relatively prime then $b \equiv$ .... (mod m).

2 Attempt any **two** :

(i) Prove that any integer > 1 can be uniquely expressed as a product of primes. **7**

(ii) Write the statement of division algorithm and prove it. **7**

(iii) State and prove Euler's theorem. **7**

3 All are Compulsory :

(i) Find the smallest positive integer x such that the remainder is 10 when it is Divided by 11, the remainder is 12 when it is divided by 13 and the remainder is 6 when it is divided by 7. **4**

(ii) Find all solutions of $x^2 \equiv 1 (mod 15)$ **6**

(iii) If $n \neq 0$, a, x, y are integers then prove that **4**

$ax \equiv ay (\text{mod } m)$ if and only if $x \equiv y \ (\text{mod } \frac{n}{(a,n)})$.

OR

3 All are compulsory :

(i) Suppose f(x) is a polynomial with integer coefficients, p is a prime number and $f(x) \equiv 0 (\text{mod } p)$ has degree n. Prove that $f(x) \equiv 0 (\text{mod } p)$ has atmost n solutions in any complete residue system(mod p). **7**

(ii) First find the solutions of $f(x) \equiv 0 (\text{mod } 3)$, $f(x) \equiv 0 (\text{mod } 5)$, $f(x) \equiv 0 (\text{mod } 7)$ and use them to find all solutions of $f(x) \equiv 0 (\text{mod } 105)$. Here $f(x) = x^4 - 1$. **7**

4 Attempt any **two** :

(i) Determine which of the following have primitive roots and if an integer has a primitive root then find at least two primitive roots : 7, 11, 12 and 35. **7**

(ii) If $\alpha \geq 3$ then prove that the set $\{5, 5^2, 5^3, \ldots\ldots 5^{2^{\alpha-2}}\}$ **7**

$\cup \{-5, -5^2, -5^3, \ldots\ldots -5^{2^{\alpha-2}}\}$ is a reduced reside system (mod 2^{α}).

(iii) Prove that $\sum_{din} \phi(d) = n$, for any positive integer n. **7**

5 Do as directed : (Each carries **2** marks)

(i) Write the statement of mobius inversion formula.

(ii) Find the value of $\varnothing(36)$ using mobius inversion formula.

(iii) Find the highest power of 11 which divides 2017!.

(iv) Find the number of positive divisors of 1660.

(v) Find the values of $\omega(n)$ for n = 25, 35, 101, 105.

(vi) Give an example of a multiplicative function which is not totally multiplicative.

(vii) Find $\varnothing(n)$ for n = 150, 307 and 19610.